Imaging of Longitudinal Electron Focusing by Light-Induced Carrier Excitation

M. Primke, J. Heil, A. Böhm, A. Gröger, and P. Wyder
Hochfeld-Magnetlabor, Max-Planck-Institut für Festkörperforschung and Centre National de la Recherche Scientifique, B.P. 166, 25 Avenue des Martyrs, F-38042 Grenoble Cedex 9, France
(Received 3 July 1996; revised manuscript received 29 January 1997)

We report on the two-dimensional observation of electron focusing under the influence of a longitudinal magnetic field $B_l$ using a real space imaging technique. As a function of $B_l$ the focusing patterns are modified and shrink together in good qualitative agreement with calculated focusing patterns of ballistic electrons. At low fields we observed Sondheimer oscillations periodic in $B_l$. At high fields light-induced magneto-oscillations due to Landau quantization occur. [S0031-9007(97)04760-1]

PACS numbers: 72.15.Eb, 03.65.Sq, 72.15.Jf, 75.80.+q

The electronic mean free path $l'$ in metals at room temperature is of the order of nm and therefore in the range of interatomic distances [1]. In very pure single crystals at low temperatures $T$, $l'$ can reach values of several 100 $\mu$m. Under these conditions it is possible to perform experiments in the so-called ballistic regime, where $l'$ is of the order of typical sample dimensions.

Sharvin [2], for example, fixed two point contacts (PC) on opposite surfaces of a Sn sample. One PC (emitter) was used to inject carriers into the crystal and the other PC (collector) detected the collector voltage $V_c$. For certain values of a longitudinal magnetic field $B_l$ (along the line connecting emitter and collector, Sharvin geometry) he observed peaks in $V_c$, a phenomenon which he called longitudinal electron focusing.

We recently presented a new experimental technique which allows one to image the far field radiation pattern of a carrier point source [3–6]. The real space imaging capability makes this technique the ideal tool for studies of the spatial distribution of the trajectories of carriers under the influence of a magnetic field, for example in the Sharvin geometry. These Sharvin-type experiments using this new technique are presented here for the first time.

The experimental setup presented in Fig. 1(a) is similar to the one described in [3,6]. The beam of a 30 mW Ar laser is coupled into an optical fiber which illuminates a small spot of the surface of a Bi single crystal slab of thickness $d \approx 230 \mu$m at $T \approx 1.5$ K, are shown in Fig. 2. $V_c$ is presented in gray scale as a function of the fiber position for different values of $B_l$ such that the electron signal appears bright.

The observation of the focusing pattern for $B = 0$ T presented in Fig. 2(a) and the underlying mechanism have...
The image frame is approximately 2.8 mT. The field dependence of \( V_c \) is shown in gray scale as a function of the fiber position. The result at low magnetic fields is presented in Fig. 2. As expected the range of \( V_c \) is confirmed by the experimental results presented in Fig. 2. As a general trend the range of \( V_c \) is suppressed. For longitudinal magnetic fields, however, the electronic contributions are expected to remain visible even at high magnetic fields. This expectation is confirmed by the experimental results presented in Fig. 2. As a general trend the range of \( V_c \) is approximately parabolic bands, which are observed in the center of the focusing pattern. In addition to this electronic signal, three dark lines emerge from the center in the directions between the bright lines. This dark structure is related to phonons [3].

In the case of rather small transverse magnetic fields (parallel to the sample surface) of the order of 1 mT the contributions of ballistic electrons are already completely suppressed. For longitudinal magnetic fields, however, the electronic contributions are expected to remain visible even at high magnetic fields. This expectation is confirmed by the experimental results presented in Fig. 2. As a general trend the range of \( V_c \) increases with \( B \) from 27 nV \( \leq V_c \leq 90 \) nV for \( B = 0 \) T to \(-920 \) nV \( \leq V_c \leq 540 \) nV for \( B = 75 \) mT. All experimental focusing patterns are scaled to the minimum (black) and maximum (white) values of \( V_c \).

Because of the symmetry of the high field focusing patterns we expected to obtain an alignment of the collector and the source with the magnetic field by placing the fiber in the center of the patterns. Here an experiment similar to that of Sharvin [2] was performed by measuring \( V_c \) as a function of \( B_l \). However, in our experiment light induced carrier excitation was used instead of carrier injection. The result at low magnetic fields is presented in Fig. 3(a). The field dependence of \( V_c \) is approximately symmetric with a maximum at \( B = 0 \) T. Peaks roughly equally spaced in \( B_l \) are observed. For higher fields \( V_c \) takes large negative values.

Figure 4 shows the theoretical focusing patterns calculated by a Monte Carlo simulation using the semiclassical model of electron dynamics, as proposed in [4]. For the calculations the so-called ellipsoidal-parabolic band structure approximation

\[
e(k) = \frac{\hbar^2}{2m} (A_x k_x^2 + A_y k_y^2 + A_z k_z^2 + 2A_{xy} k_x k_y)\]

is used with \( A_x = 202, A_y = 1.67, A_z = 70, A_{xy} = 7.0 \) [8]. The Fermi energy is \( \varepsilon_F = 17.7 \) meV and \( m \) is the free electron mass. The advantage of this simplified band structure approximation is that the semiclassical equations of motion [1] for a uniform magnetic field \( B_z \) can be

---

**FIG. 2.** Experimental focusing patterns for different values of the longitudinal magnetic field: The voltage \( V_c \) at the collector is shown in gray scale as a function of the fiber position. The image frame is approximately \( 450 \mu m \). (a) \( B = 0 \) T; (b) \( B = 1.1 \) mT; (c) \( B = 1.7 \) mT; (d) \( B = 2.3 \) mT; (e) \( B = 2.8 \) mT; (f) \( B = 4.5 \) mT; (g) \( B = 25 \) mT; (h) \( B = 50 \) mT; (i) \( B = 75 \) mT.

**FIG. 3.** (a) Light-induced longitudinal electron focusing in Bi. The voltage \( V_c \) is measured as a function of a longitudinal magnetic field \( B_z \). The de Haas–van Alphen frequencies for holes \( F_{h} \) and electrons \( F_{e} \) as a function of a magnetic field \( B \) approximately parallel to the trigonal axis of Bi at three different positions of excitation. The gray scale images show the corresponding electron focusing patterns at \( B = 0 \) T (\(-8.1 \) nV \( \leq V_c \leq -0.3 \) nV) and \( B = 30 \) mT (\(-840 \) nV \( \leq V_c \leq 285 \) nV) with an image frame of \( 450 \) \( \mu m \). (c) Fourier spectra of the \( V_c(1/B_z) \) curves of (b). The de Haas–van Alphen frequencies for holes \( F_{h} \) and electrons \( F_{e} \) are indicated with the possible range for \( F_{e} \) for a tilt of the field with respect to the trigonal axis of the crystal up to \( 5^\circ \) as a shaded horizontal bar.

---
Theoretical focusing pattern for different values of the longitudinal magnetic field. The image frame is (400 \mu m)^2 and the sample thickness is \( d = 200 \mu m \). The fields correspond to those shown in Figs. 2(a)–2(f). (a) \( B = 0 \) T; (b) \( B = 1.1 \) mT; (c) \( B = 1.7 \) mT; (d) \( B = 2.3 \) mT; (e) \( B = 2.8 \) mT; (f) \( B = 4.5 \) mT. The patterns are calculated with 10^7 events for \( B = 0 \) T and with 10^8 events for \( B \neq 0 \) T using the model described in [4].

Some calculated electron trajectories for different values of \( k_z \) and \( B_z \) are presented in Figs. 1(c)–1(h).

Longitudinal focusing occurs when the electrons reach the opposite surface after an integer number \( n \) of revolutions. The condition can be obtained by solving the semiclassical equations of electron dynamics, or by using Sharvin’s focusing condition [2] that the propagation \( g \) along a magnetic field in \( z \) direction during one revolution is

\[
g = \frac{h}{eB_z} \left( \frac{\partial S_k}{\partial k_z} \right)_{\epsilon=\text{const}}, \tag{2}
\]

where \( S_k \) is the area of intersection of the Fermi surface with a plane \( k_z = \text{const} \).

For the band structure in (1) both approaches lead to the longitudinal focusing condition:

\[
B_{loc} = n \frac{k_z}{d} \frac{2\pi \hbar}{e} \left( A_{z} - A_{z}^2 \right) \left( A_x A_y \right)^{-1/2}. \tag{3}
\]

This is equivalent to the condition for oscillations of the Sondheimer effect (d.c. size effect) [9]. Here the contributions for different values of \( k_z \) are reported to interfere destructively, so that only feeble oscillations survive, with \( B_{loc}^* \) that corresponds to a limiting wave vector \( k_z^* \) of the Fermi surface [9]. For the band structure in (1) we obtain \( k_z^* = 1.08 \times 10^8 \) m\(^{-1} \) which gives \( B_{loc}^* = n \times 4.8 \) mT.

In the following we discuss the modifications of the focusing patterns under the influence of a longitudinal magnetic field. For increasing \( B_{l} \), a second set of bright lines parallel to the first one appears [see Figs. 2(c) and 2(d)]. A similar behavior is observed in the theoretical focusing patterns in Figs. 4(c) and 4(d). For higher fields, the theoretical focusing patterns shrink together due to the scaling of the real space trajectories with \( 1/B_{l} \).

The corresponding bright spot can be observed in the center of symmetry of the experimental focusing pattern in Fig. 4(f).

The dark shadow region of Fig. 2(e), which shrinks together to a point for increasing \( B_{l} \) as in Fig. 2(i), may be interpreted as a diffuse contribution of the holes which surmounts the electronic signal for \( B_{l} > 25 \) mT. For \( B_{l} < 25 \) mT the hole contribution is expected to be small because of two reasons: First, the hole ellipsoid in the Fermi surface [see Fig. 1(b)] is not stretched in the same way as the electron ellipsoids, which results in a smooth, but not focused contribution over the whole pattern. Second, the mean free path of holes in Bi is much smaller than that of the electrons [10], which leads to a faster decay of the hole contribution with distance. With increasing field this behavior changes: If \( B_{l} \) is sufficiently high, even for a large number of scattering processes in the bulk, the hole trajectories are restricted to a narrow cone around the field axes, having a common origin in the illuminated spot. The opening angle of this cone increases with the number of scattering processes and decreases with increasing \( B_{l} \). The dominance of the hole signal over the electronic signal at high \( B_{l} \) is presumably due to a phonon drag effect. Already the pattern at \( B = 0 \) T indicates that the coupling of the phonons to the holes is stronger than to the electrons, because the phonon signal (dark structure) has the opposite sign with respect to the electron signal (bright).

The origin of the fanlike structure observed around the dark spot at high fields [Fig. 2(g)–2(i)] is not yet understood. It is remarkable that the pattern evolves from threefold symmetry for \( B = 0 \) T to sixfold symmetry for high fields. The different modes of calculated phonon focusing patterns do not correspond to this fanlike structure. Because of its extension, it cannot be related to purely ballistic carrier propagation, but may be some diffusive process, probably related to the anisotropy of the magnetoresistance.

From the observations mentioned above, the \( V_r(B_{l}) \) characteristic in Fig. 3(a) can be interpreted as being composed of at least two contributions. The first one describes the general behavior due to a concentration of diffusive hole flux onto the collector of finite area. Since the trajectories scale in the \( xy \) plane with \( 1/B_{l} \), one should expect \( V_r \) to be quadratic in \( B_{l} \), which is in good qualitative agreement with the behavior of the signal in Fig. 3(a). In addition, the signal shows oscillations roughly periodic in \( B_{l} \) and the period corresponds approximately to the theoretical value \( B_{loc}^* = n \times 4.8 \) mT of Sondheimer oscillations.

The \( V_r(B_{l}) \) characteristics at high magnetic fields are shown in Fig. 3(b) for different positions of the illuminated spot with respect to the collector point contact. The positions are indicated in the inset of the focusing pattern for \( B = 30 \) mT, recorded on a Bi single crystal slab of the thickness \( d = 400 \mu m \). These patterns are similar to those of Fig. 2 but they do not show the dark.
spot expected at higher fields, probably due to a small
tilt between the crystal and the field axis. Pos E, Pos H
and Pos O are situated near the maximum, minimum, or
correspondingly zero value of $V_c$ in the focusing pattern
for $B = 30$ mT. Within our convention for $V_c$ used in
the gray scale patterns, the electron signal appears bright
(Pos E) and the hole signal is shown dark (Pos H).

All measured $V_c(B)$ curves (only three are presented
here) show magneto-oscillations periodic in $1/B$, on a
background which is increasing to a maximum value at
$B = 0.3$ T and with a strong decay towards high fields.

The Fourier amplitude spectra of the $V_c$'s depicted in
Fig. 3(c) show peaks for certain values of the de Haas-van
Alphen frequency $F$. Since the Fermi surface of Bi
consists of three electron ellipsoids and one hole ellipsoid [see Fig. 1(b)], there should be at the
most four frequencies and their harmonics. For $B$ exactly
along the trigonal axis the three electron frequencies are
degenerated ($F_e = 8.72$ T and $F_h = 6.34$ T [9]). The
range of possible frequencies of electrons for a deviation
of $B$ of up to $5^\circ$ with respect to the trigonal axis (this is
our worst case estimate) are sketched in the Fourier plots
as a shaded bar. For the holes, $F_h$ is the minimum value
and the variations for a tilt within $5^\circ$ are small [9]. The
Fourier amplitude of the sweep in Pos H shows an intense
peak near $F_h$ and additional peaks at the first and second
harmonics. At Pos E the peak at $F_h$ is reduced but another
peak develops at $F = 10.3$ T that could be interpreted as
one of the electron peaks for a small tilt between the field
and the trigonal axis. Since all the high field sweeps are
recorded with the same field orientation, the frequencies
of the Fourier peaks should be the same. However, we
observed the nonexpected result that the amplitudes of the
peaks depend on the position of the excitation with respect
to the collector PC.

In conclusion, the strong influence of a longitudinal
magnetic field on the carrier propagation emerging from
a pointlike source has been demonstrated by using a
real space resolving scanning technique. Proceeding from
the established situation at zero magnetic field, several
new effects have been shown for the first time by light-
induced carrier excitation. Longitudinal electron focusing
was observed by reproducing the Sharvin experiment with
light-induced carrier excitation instead of carrier injection
through a point contact.

At low magnetic fields we observed Sondheimer oscil-
lations. With our real space scanning technique it is pos-
sible to identify the contributions of electrons and holes
due to the evolution of their focusing patterns in a lon-
gitudinal magnetic field. Additional contributions that
are probably related to the anisotropic magnetoresistance
were observed at magnetic fields of several 10 mT. At
high magnetic fields we observed light induced magneto-
oscillations due to Landau quantization with an amplitude
depending on the position of excitation. The further de-
velopment of this technique could lead to deeper insights
into the physics of magneto-oscillations by the possibility
of their systematic spatially resolved observation. More-
over, in the context of the earlier results already obtained
using this innovative technique, the new observations pre-
sented in this Letter underline the potential of the method
and its evolution in the future.

[1] W. Ashcroft and D. Mermin, Solid States Physics (Saun-
54 (1965) [JETP Lett. 1, 152 (1965)].
[9] D. Shoenberg, Magnetic Oscillations in Metals (Cam-
(1993).